Indoor positioning using Particle Filter

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Contents

[1 Introduction 2](#_Toc506237899)

[2 Trilateration [1] 2](#_Toc506237900)

[3 Particle Filter 3](#_Toc506237901)

[3.1 Problem formulation 3](#_Toc506237902)

[3.2 The need: Estimate conditional probability- 4](#_Toc506237903)

[3.3 Bayesian Recursion 4](#_Toc506237904)

[3.4 Grid based discrete probability distribution approximation 4](#_Toc506237905)

[3.5 Grid based continuous probability distribution approximation 4](#_Toc506237906)

[3.6 Basic algorithm 4](#_Toc506237907)

[3.7 Resampling 4](#_Toc506237908)

[3.8 The choice of 4](#_Toc506237909)

[4 Application of PF to Indoor Positioning [4] 4](#_Toc506237910)

[4.1 State equation 4](#_Toc506237911)

[4.1.1 Known acceleration 4](#_Toc506237912)

[4.1.2 Unknown acceleration 5](#_Toc506237913)

[4.2 6](#_Toc506237914)

[4.3 Measurement equation 6](#_Toc506237915)

[4.4 6](#_Toc506237916)

[4.5 Initial state [4] 6](#_Toc506237917)

[4.5.1 Least Square estimation 6](#_Toc506237918)

[4.5.2 Initial state particles 7](#_Toc506237919)

[5 Simulations 7](#_Toc506237920)

[6 References 9](#_Toc506237921)

# Introduction

This report summarizes the short investigation of the indoor positioning problem faced by Tracxpoint and suggests a possible solution; the Particle Filter . The chosen technology is the Ultra Wideband (UWB) and the device is the almost industry standard Decawave’s DW1000.

# Trilateration [1]

The basic information provided by the DW1000 is the Time Of Flight (TOF) from a given tag to an anchor. This time of flight is assumed to reflect the distance according to:

|  |  |  |
| --- | --- | --- |
|  |  | ‎2.1 |

Where; , is the speed of light, and is the anchor’s index.

The basic positioning derivation technique is the Trilateration; TOF between a tag and 3 or more anchors allows to intersect 3 imaginary spheres having each a radius . In fact, since the heights of both tag and anchor are known and constant, we may conduct all the calculation in a 2-dimensional plane (Pythagorean triplet);

|  |  |  |
| --- | --- | --- |
|  |  | ‎2.2 |

Where;

|  |  |  |
| --- | --- | --- |
|  |  | ‎2.3 |

An illustration of the intersection is brought in Figure 1;

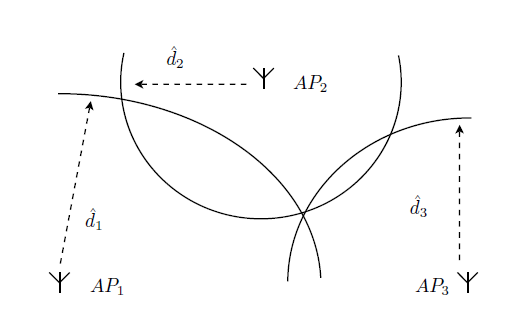


Figure : Trilateration

The Trilateration is a “memoryless” technique, as it uses only the current observation to derive the location. Another memoryless technique exists, Multilateration, but is much less used for the reasons mentioned in [2].

# Particle Filter

## Problem formulation

The Particle Filter has been widely used in motion tracking problems, whose observations are noisy. In our case, the estimated parameter is the current position and velocity of an object, the state vector, and the observations are the distances (or the square distances) to the anchors. In general, the state and the observation are vectors. We regard the state and its observation as a Hidden Markovian Random Process (HMM):

…

Figure : HMM process model

Hence, known algebraic and probabilistic relationships between current state and observation to the previous.

Probabilistic relationship;

|  |  |  |
| --- | --- | --- |
|  |  | ‎3.1 |

The physical model assumed by the Kalman filter is expressed by the algebraic relationship:

|  |  |  |
| --- | --- | --- |
| The State equation |  | ‎3.2 |

|  |  |  |
| --- | --- | --- |
| The Measurement equation |  | ‎3.3 |

Where;

* is the previous step’s added noise; the state equation noise.
* is the current step’s added noise; the measurement equation noise.

The functions are not necessarily linear, and the noises are not necessarily Gaussian thus unfit to be treated by the Basic or the Extended Kalman filter.

## The need: Estimate conditional probability-

The particle filter aims to estimate the conditional probability and further use it to generate 2 possible types of estimators:

* MAP estimator:
* MMSE estimator:

This will be done in a sequential manner

## The Particle principle: Grid-based approximation

We intend to approximate the continuous probability distribution mentioned in ‎3.2 based on observations (“Monte Carlo”). The grid based approximation suggests that a discrete -long grid of vectors belonging to the continuous probability space be sampled (to become a grid) and that the required distribution be evaluated on that grid. Namely, is the continuous -dimensional vector for which we define;

The grid at step ;

The conditional probability at step on the grid;

And the probability is approximated as follows;

The terms of the grid are called “Particles”, and the conditional probabilities are called “Weights”.

The Particle Filtering algorithm is a sequential procedure of;

* Particle vector realization (based on a known probability distribution predefined by the model)
* Observation acquisition
* Weights calculation

## Bayesian Recursion

The basis of the sequential algorithm is the Bayesian recursion:

* Prediction:
* Update:

Where;

## Grid based discrete probability distribution approximation

## Grid based continuous probability distribution approximation

## Basic algorithm

## Resampling

## The choice of

# Application of PF to Indoor Positioning [3]

## State equation

In the indoor positioning case, the state, is a vector;

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.1 |

Where are the 2D position and velocity respectively.

### Known acceleration

The state equation describes an accelerated movement, where the 2D acceleration is known in principle;

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.2 |
|  |  |  |

And is thus linear:

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.3 |

The acceleration is treated as an input and is supposed to be provided by an acceleration sensor (accelerometer). The noise is modeling the accelerometer inaccuracy and is supposed to be white;

Hence;

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.4 |

The value of and should be provided by the accelerometer device.

### Unknown acceleration

When the acceleration is unknown, we can regard it as a white random process. The state equation in this case is;

Whose noise vector is:

The noise covariance matrix is:

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.5 |

and are and respectively (). A good choice would be the maximum possible acceleration on each of the axes. The null terms in the matrix are the result of lack of correlation between and .

## 

## Measurement equation

The observation vector;

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.6 |

And the observation equation is;

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.7 |

Which is obviously nonlinear.

The noise is modeling the TOF measurement inaccuracy and is presumed white;

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.8 |

## 

## Initial state [3]

### Least Square estimation

The EKF is a sequential algorithm, and it is therefore useful to have an initial state that is as accurate as possible. One option is to solve the nonlinear system ‎2.3. It is also possible to formulate a linear least squares problem (true for all , namely for );

Which becomes;

And in matrix form;

Solving for , we may derive to give (assuming initial velocities are zero):

|  |  |  |
| --- | --- | --- |
|  |  | ‎4.9 |

### Initial state particles

# Simulations

We have a recording of a moving tag along a known grid ( between each grid); ranges from 3 anchors sampled at a interval.

We have estimated the track using Trilateration and PF. The PF supposed unknown acceleration. The PF’s model tunable parameters are;

* The measurement noise: (measured in units of )
* The state noise: (measured in units of )
* The particles number:
* The resampling threshold:

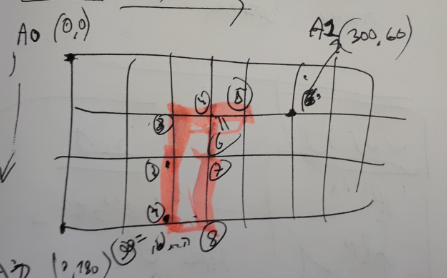
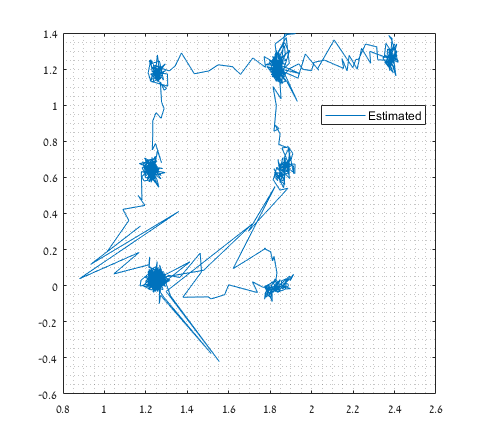


Figure : Ground Truth. Grid=

Figure : Trilateration

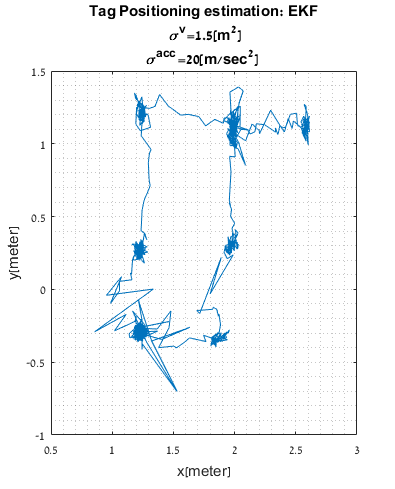
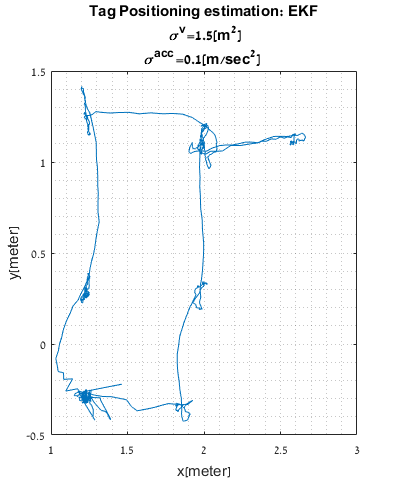


Figure : EKF- performance versus state noises/acceleration. Large state noise (left), small state noise (right)

We may notice the large fluctuations in the trilateration case comparing to the EKF case. Also, we detect the resemblance of the EKF with large state noise to the trilateration. This makes sense, as the noisier the previous state the less we rely on it on estimating the current state and the more on the current measurement; which is exactly what the trilateration does

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